

Universal AI: AIXI

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Iliad Intensive

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- **The common engine**: a single **Bayesian mixture** ξ over a class $\mathcal{M}$ of candidate environments, weighted by a prior w_ν .
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- **Universal** choice: $\mathcal{M}$ = all computable environments, prior $w_\nu = 2^{-K(\nu)}$ (K is the “complexity” of the environment). Assumption “ $\mu \in \mathcal{M}$ ” becomes “*the universe is computable*”.

The Bayesian mixture ξ

Definition

Class $\mathcal{M} = \{\nu_1, \nu_2, \dots\}$ with prior weights $w_\nu > 0$, $\sum_\nu w_\nu = 1$.

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- **Key fact (mixture is a predictor):** its one-step prediction is a *posterior-weighted* average of the members' predictions (you prove this today!)

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- **Posterior update is multiplicative:** $w(\nu \mid x_{1:t}) = w(\nu \mid x_{<t}) \cdot \frac{\nu(x_t \mid x_{<t})}{\xi(x_t \mid x_{<t})}$: we reweight by how well ν predicted vs. ξ (Bayes' rule!)

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- Spaces: actions $\mathcal{A}$, observations $\mathcal{O}$, rewards $\mathcal{R} \subset [0, 1]$, percepts $\mathcal{E} = \mathcal{O} \times \mathcal{R}$, and (finite) histories $\mathcal{H}^* := (\mathcal{A} \times \mathcal{E})^* := \cup_{t=0}^{\infty} (\mathcal{A} \times \mathcal{E})^t$.

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- **No Markov / MDP assumption!** Both the policy and environment can depend on all prior percepts.

The three players

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Interaction measure ν^π

Run π inside ν and multiply along the history:

$$\nu^\pi(\mathfrak{a}_{t:m} \mid \mathfrak{a}_{<t}) := \prod_{k=t}^m \pi(a_k \mid \mathfrak{a}_{<k}) \nu(e_k \mid \mathfrak{a}_{<k} a_k).$$

Value, optimality, and AIXI

Value function (assuming geometric discount γ)

$$\begin{aligned} V_{\nu}^{\pi,m}(\mathfrak{a}_{<t}) &= (1 - \gamma) \mathbb{E}_{\nu}^{\pi}[G_{t-1:m} \mid \mathfrak{a}_{<t}] \text{ with } G_{t-1:m} = \sum_{k=t}^m \gamma^{k-t} r_k \\ &= (1 - \gamma) \sum_{\mathfrak{a}_{t:m}} \nu^{\pi}(\mathfrak{a}_{t:m} \mid \mathfrak{a}_{<t}) \sum_{k=t}^m \gamma^{k-t} r_k \end{aligned}$$

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- **Optimal value** $V_{\nu}^{*} := \sup_{\pi} V_{\nu}^{\pi}$; an optimal policy π_{ν}^{*} satisfies $V_{\nu}^{\pi_{\nu}^{*}} = V_{\nu}^{*}$.
- **Bayes-optimal policy** $\pi_{\xi}^{*} \in \arg \max_{\pi} V_{\xi}^{\pi}$: acts optimally w.r.t. the *mixture* ξ .

AIXI

π_{ξ}^* with $\mathcal{M}$ = all lower-semicomputable environments and prior $w_{\nu} = 2^{-K(\nu)}$. Write ξ_U for ξ with this choice of $\mathcal{M}$ and w_{ν} , so $\text{AIXI} = \pi_{\xi_U}^*$.

Intuition: *The Bayes-optimal agent with prior according to Occam's razor: Act optimally according to environments that you believe to be in, assuming the simplest environments consistent with observations are more likely to be true.*

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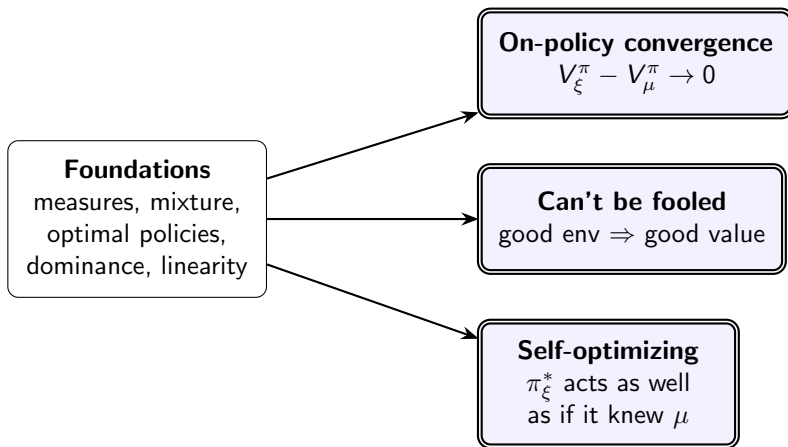
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AIXI, explicitly

max over actions, $\sum$ over percepts, choosing actions that maximise the ξ -weighted discounted return:

$$a_t = \underset{a_t}{\text{arg max}} \lim_{m \rightarrow \infty} \sum_{e_t} \underset{a_{t+1}}{\text{max}} \sum_{e_{t+1}} \cdots \underset{a_m}{\text{max}} \sum_{e_m} \xi(e_{t:m} \mid \mathfrak{a}_{<t} a_{t:m}) \sum_{k=t}^m \gamma^{k-t} r_k$$

The three main AIXI results



- All three: the Bayesian agent inherits the guarantees it would have if it knew the truth.

Result 1: On-policy value convergence

Theorem

For any $\mu \in \mathcal{M}$ and any policy π ,

$$V_{\xi}^{\pi}(\mathfrak{a}_{<t}) - V_{\mu}^{\pi}(\mathfrak{a}_{<t}) \rightarrow 0 \quad \mu^{\pi}\text{-almost surely.}$$

Intuition: *A policy that acts, pretending the environment is ξ , will eventually act as well as if it were in the true environment μ .*

Result 2: AIXI can't be fooled

Theorem (deterministic μ)

If the truth is reachable-good, $V_\mu^*(\mathfrak{a}_{<t}) > \varepsilon$ along the played history, then

$$V_\xi^*(\mathfrak{a}_{<t}) \geq w_\mu \varepsilon > 0.$$

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Intuition: *Whenever good behaviour in μ is achievable, the Bayes-optimal agent secures at least a fraction proportional to the prior weight of μ .*

Result 3: Self-optimizing property (stretch goal)

Theorem

If *some* policy can learn to act optimally in every $\nu \in \mathcal{M}$, then π_ξ^* does too:

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Intuition: *The Bayes-optimal agent that is unaware of which environment it is in learns to act just as well, as the best possible policy that already knows which environment it is in (μ).*

Takeaways

- **Solomonoff** solves induction: mix over all computable hypotheses, weighted by simplicity. Total prediction error $\leq K(\mu) \ln 2$.

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- **Why care for safety:** a precise, assumption-light model of idealized intelligence providing a lens on what optimal agents do. For example, we can show under what circumstances AIXI would [wirehead](#) [RO11], [safely rather than recklessly explore](#) [CCH20], or [self-modify](#) [OR11] as a model of what superintelligent agents would do.

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